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REAL-TIME PREDICTION OF NEAR-FUTURE SEISMIC EXCITATION ADAPTING AR MODEL TO PRECEDING INFORMATION

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SUMMARY

This paper examines a real-time prediction method, aimed at application in active structural control. The examined method applies preceding seismic excitation information at a certain moment to a time-variant AutoRegressive (AR) model and uses it to predict near-future excitation information. The performances of this method and appropriate identification parameters are examined by numerical experiments. In fact, the results of these experiments show that a time-variant AR model with appropriate identification parameters has little change in low-frequency components despite change in AR coefficients. The performance of a fixed-coefficient AR model is thus examined. The results show that even a fixed-coefficient AR model can sufficiently predict 0·05-s-future excitation information. Copyright © 1999 John Wiley & Sons Ltd.

KEY WORDS: parameter identification; seismic excitation; real-time prediction; autoregressive model

INTRODUCTION

Seismic excitations are neither random nor stationary. Several waves such as P-waves, S-waves and surface waves appear during an earthquake. The Fourier transfer shows the extent of a certain frequency component in a seismic excitation. A response spectrum tells us how much a seismic excitation influences a structure at each fundamental period. We have used these methods to estimate design-seismic loads for structures.

We can more positively consider the influence of seismic excitations in active structural control. If we can estimate near future information in advance, we can incorporate feedforward effects into a control law or develop a predictive-adaptive control law using excitation information updated at each moment. We can thus expect more reduction and efficiency, which cannot be achieved by feedback control laws. The instantaneous power spectrum,[1] the evolutionary spectrum,[2] the physical spectrum,[3] the phase spectrum,[4] the stochastic-process application,[5] the wavelet transfer[6] and so on are proposed to analyse time-varying characteristics of a seismic excitation in detail. However, it is inconvenient to reflect the results obtained by these methods in active

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structural control. Some of these require full or large number of records for analyses. Some of these are not adaptable to a real-time prediction of a seismic excitation. Information in the time domain is preferable to that in the frequency domain because most modern control laws are constructed in the time domain.

If a state equation model expresses excitation information, it can be easily combined with a control law. Furthermore, the state equation model can become a predictor by providing information time by time. Examples of state equation models include an Autoregressive Moving Average (ARMA) model or more simply an Autoregressive (AR) model (e.g. Reference 7). These models are easy to handle because many methods are proposed to identify their coefficients from a series of digital signals. In fact, Nau *et al.*,[8] Samaras *et al.*,[9] Hoshiya and Maruyama,[10] Nishide *et al.*,[11] Matsumura and Tomizawa,[12] Ólafsson and Sigbjörnsson,[13] and Kawakami and Bidon[14] showed that the AR and ARMA models can be applied to seismic excitations. In addition, Kanda *et al.*[15] showed that one of these methods can provide future excitation information. The author and Kobori[16,17] also showed that a time-variant AR model can predict near future excitation; the algorithm proposed by Burg[18] and Anderson[19] is practical for identifying its coefficients, and active control laws can combine it. This paper examines its performances in greater detail.

In the following, the time-variant AR model is first introduced and it is related to the state equation form. Then its transfer function and decomposed form are introduced. Next, its basic performance and efficiency as a predictor are examined by applying it to sine and seismic excitations. Appropriate parameters for identifying coefficients of the time-variant AR model are also determined. In fact, the transfer function and decomposed form of the time-variant AR model show that its low-frequency components change little when appropriate parameters are chosen. Thus, the performance of a fixed-coefficient AR model is also examined.

TIME-VARIANT AR MODEL

State equation: For the digital signals recorded by sampling time Δt, let step r mean time $r\Delta t$. Then, apply a time-variant AR model to a seismic excitation as

$$w(r|k) = -a_{1|k}w(r-1|k) - a_{2|k}w(r-2|k) - \cdots - a_{q|k}w(r-q|k) + \zeta(r|k) \qquad (1)$$

where $w(r|k)$ for $r \geqslant k$ is the excitation information at r estimated at k; $a_{1|k}, \ldots, a_{q|k}$ are called AR coefficients and determined at k using the information from $k-l+1$ to k and successively updated; and $\zeta(r|k)$ indicates an identification error, which should be a Gaussian process. Burg's algorithm[18] is effective in identifying the AR coefficients.

Using the z-transfer, frequency characteristics of the excitation $\tilde{w}(z|k)$ are expressed by

$$\tilde{w}(z|k) = \left[1 + \sum_{j=1}^{q} a_{j|k} z^{-r}\right]^{-1} \tilde{\zeta}(z|k) \qquad (2)$$

Then, the transfer function of the time-variant AR model $H(\omega|k)$ is defined by

$$H(\omega|k) = \left[1 + \sum_{j=1}^{q} a_{j|k} \exp(-\mathrm{i}\omega\Delta t)\right]^{-1} \qquad (3)$$

where ω is circular frequency and $\mathrm{i} = \sqrt{-1}$.

The AR model can be converted to the state and observed equations whose state vector $v(r)$ consists of excitation information from step $r-q$ to step $r-1$. That is,

$$v(r+1\,|\,k) = \mathbf{H}_{|k} v(r\,|\,k) + \mathbf{L}\zeta(r\,|\,k) \tag{4}$$

$$w(r\,|\,k) = \mathbf{F}_{|k} v(r\,|\,k) + \zeta(r\,|\,k) \tag{5}$$

where

$$v(r\,|\,k) = \begin{Bmatrix} w(r-q\,|\,k) \\ w(r-q+1)\,|\,k) \\ \cdots \\ w(r-2\,|\,k) \\ w(r-1\,|\,k) \end{Bmatrix}, \qquad \mathbf{H}_{|k} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_{q\,|\,k} & -a_{q-1\,|\,k} & -a_{q-2\,|\,k} & \cdots & -a_{1\,|\,k} \end{bmatrix},$$

$$\mathbf{L} = \begin{Bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 1 \end{Bmatrix}$$

$$\mathbf{F}_{|k} = -\left[a_{q\,|\,k}\ \ a_{q-1\,|\,k}\ \ a_{q-2\,|\,k}\ \ \cdots\ \ -a_{1\,|\,k}\right] \tag{6}$$

Thus, $v(r\,|\,k) \in \mathbf{R}^q$, $w(r\,|\,k) \in \mathbf{R}$, $\zeta(r\,|\,k) \in \mathbf{R}$, $\mathbf{H}_{|k} \in \mathbf{R}^{q\times q}$, $\mathbf{L} \in \mathbf{R}^q$ and $\mathbf{F}_{|k}^{\mathrm{T}} \in \mathbf{R}^q$ where $\mathbf{R}$ is the set of all real numbers.

We can examine the AR model's characteristics by mode decomposition, letting $\mathbf{\Lambda}_{|k} = \mathrm{diag}\{\lambda_{i\,|\,k}\} = \mathbf{\Psi}_{|k}^{\mathrm{T}}\mathbf{H}_{|k}\mathbf{\Psi}_{|k}$, $\mathbf{\Psi}_{|k} = \{\,..\,\psi_{i\,|\,k}\,..\}$ and $\mathbf{\Psi}_{|k}^{\mathrm{T}}\mathbf{\Psi}_{|k} = \mathbf{I}$, i.e.

$$v_i^*(r+1\,|\,k) = \psi_{i\,|\,k}^{\mathrm{T}}\mathbf{H}_{|k}\psi_{i\,|\,k}\psi_{i\,|\,k}^{\mathrm{T}} v(r\,|\,k) + \psi_{i\,|\,k}^{\mathrm{T}}\mathbf{L}\zeta(r\,|\,k) = \lambda_{i\,|\,k} v_i^*(r\,|\,k) + \gamma_{i\,|\,k}\zeta(r\,|\,k) \tag{7}$$

$$w(r\,|\,k) = \mathbf{F}_{|k}\mathbf{\Psi}_{|k}\mathbf{\Psi}_{|k}^{\mathrm{T}} v(r\,|\,k) + \zeta(r\,|\,k) = \sum_{i=1}^{q} f_{i\,|\,k} v_i^*(r\,|\,k) + \zeta(r\,|\,k) \tag{8}$$

Let us call $v_i^*(r\,|\,k) = \psi_{i\,|\,k}^{\mathrm{T}} v(r\,|\,k)$ and $f_{i\,|\,k} = \mathbf{F}_{|k}\psi_{i\,|\,k}$ the ith elemental wave fixed at k and its contribution factor, respectively. That is, the excitation is expressed as the sum of elemental waves weighted by contribution factors. At this time

$$v_i^*(r\,|\,k) = \lambda_{i\,|\,k}^{r-k} v_i^*(k\,|\,k) + \gamma_{i\,|\,k}\sum_{j=k}^{r-1} \lambda_{i\,|\,k}^{r-1-j}\zeta(j\,|\,k) \tag{9}$$

If all the absolute values of all $\lambda_{i\,|\,k}$ are less than 1·0, the time-variant AR model is stable. The angle of $\lambda_{i\,|\,k}$ divided by Δt means the circular frequency of the ith elemental wave.

Once the AR coefficients are identified at k, the excitation at $(k+p)$ is predicted by

$$v(k+p|k) = \mathbf{H}_{|k}^{p} v(k|k) + \sum_{r=1}^{p} \mathbf{H}_{|k}^{p-r} \mathbf{L}^{r} \zeta(k|k) \tag{10}$$

$$w(k+p|k) = \mathbf{F}_{|k} v(k+p|k) + \zeta(k|k) \tag{11}$$

PERFORMANCES OF TIME-VARIANT AR MODEL

Basic performances for sine excitations: Let us first examine the basic performances when the proposed method is applied to sine excitations on the condition that the parameters for identification are fixed as $\Delta t = 0{\cdot}01$, $q = 4$ and $l = 100$. To examine the difference caused by dynamical characteristics of the excitations, let sine excitation frequencies be 0·2, 1·0 and 5·0 Hz, where dominant components of most seismic excitations are involved.

Figure 1 compares the excitations predicted using the 0·05-s-prior information with the original, and shows the AR coefficients at each moment. The predicted excitations for the sine 0·2 and 1·0 Hz excitations agree closely with the original, while those for the sine 5 Hz excitation have a phase delay and smaller amplitude. The identified AR coefficients change regularly, but each case has a different nature. For the sine 1·0 and 0·2 Hz excitations, only two AR coefficients are mostly effective. For the sine 5 Hz excitations, four coefficients are effective and have little change. From the AR coefficients, we can obtain transfer functions $H(\omega|k)$ of the time-variant AR models at each moment, as shown in Figure 2, where the axis from the centre to the right indicates frequency, the axis from the centre to the left indicates time and the vertical axis indicates amplitude. As shown in Figure 2, the changes in low frequencies are small in the transfer functions despite change in the AR coefficients and all the time-variant AR models work as low pass filters, reducing 4 Hz-or-higher-frequency components. Thus, the predicted wave for the sine 5 Hz wave does not agree closely with the original.

Performances for seismic excitations: Next, let us examine the performances for seismic excitations. The assumed excitations are El Centro 1940 NS (El Centro) and Hachinohe 1968 NS (Hachinohe). Let $\Delta t = 0{\cdot}01$ s, so that linearly compensated signals from the data at the close steps are used if a record lacks information at a sampling time. Furthermore, the performances of different-dimensional AR models are compared assuming that $l = 100$ and $q = 2$ and 4.

Figures 3 and 4 show the predicted excitations using the 0·05-s-prior information compared with the original and the identified AR coefficients for El Centro and Hachinohe. The predicted excitations using the 0·05-s-prior information mostly agree with the original for both $q = 2$ and 4. However, neither case traces the peaks from 4·0 to 5·0 s. When $q = 2$, the AR coefficients for both El Centro and Hachinohe are almost maintained at $-2{\cdot}0$ and 1·0. Thus, $w(r|k) - w(r-1|k) \approx w(r-1|k) - w(r-2|k)$, i.e. the increments of the acceleration are assumed to be approximately constant for all steps. When $q = 4$, the AR coefficients for El Centro are mostly maintained at $-1{\cdot}85$, 0·85, $-0{\cdot}2$ and 0·15, but those for Hachinohe gradually change. Figure 5 shows the transfer function of the time-variant AR models for El Centro and Hachinohe when $q = 4$. The AR model for El Centro filters high-frequency components, although it allows around 5·0 Hz components to be amplified when the wave has strong peaks. The AR model for Hachinohe at first amplifies the high frequency components, but later filters them. Thus, the peaks from 4·0 to 5·0 s are not traced.

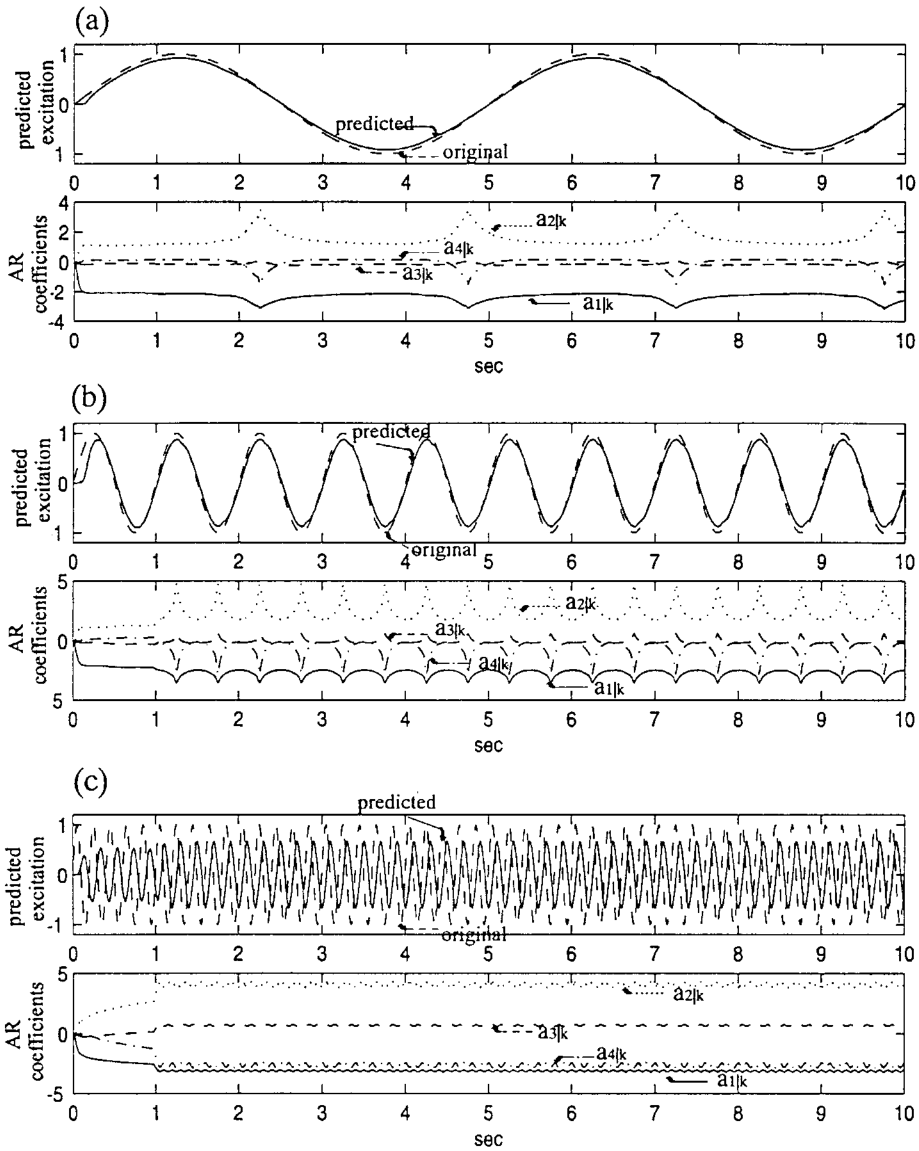

Figure 1. Predicted excitations and AR coefficients for sine excitations: (a) for sine 0·2 Hz; (b) for sine 1 Hz; (c) for sine 5 Hz

Figure 6 shows the decomposed characteristics and identification error for Hachinohe when $q = 4$. As shown in Figure 6(a), the absolute values of all eigenvalues are less than 1·0, so that the identified time-variant AR model is stable. Two of the eigenvalues are real and the others are a conjugate complex pair. The absolute values of the complex eigenvalues, called C1 and C2, are approximately 1·0, while their angles mostly vary between $\pi/9$ and $\pi/4$ or between $-\pi/9$

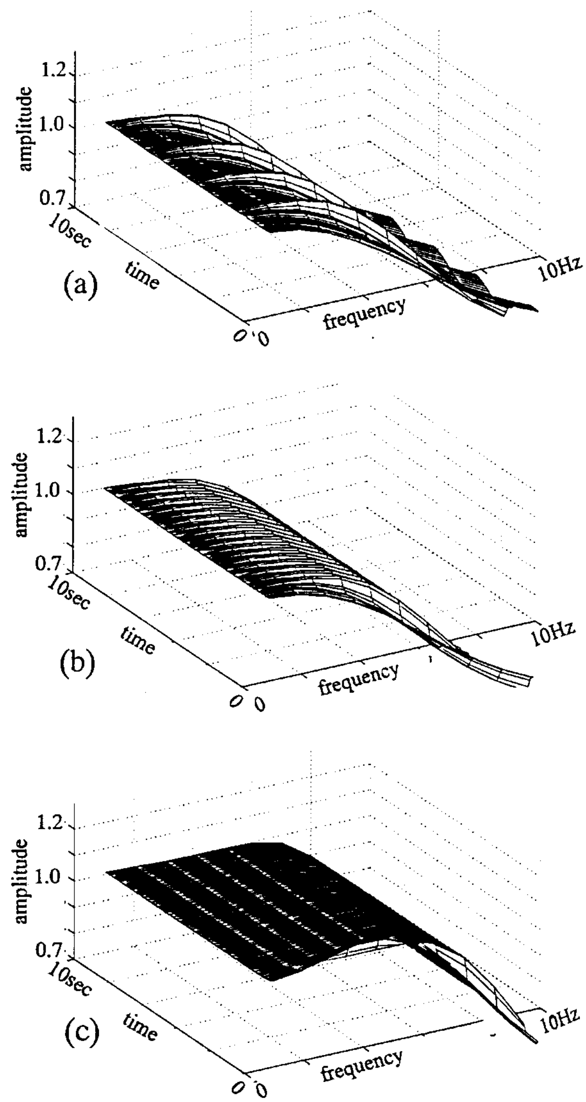

Figure 2. Transfer functions of time-variant AR model for sine excitations ($\Delta t = 0{\cdot}01$ s, $l = 100$, $q = 4$): (a) for sine 0·2 Hz; (b) for sine 1 Hz; (c) for sine 5 Hz

and $-\pi/4$, which means change in 5·0 Hz or higher-frequency components. One of the real eigenvalues, called R1, is mostly maintained at about 1·0, while the other, called R2, gradually changes. As shown in Figure 6(b), the contribution factors corresponding to R1 are about 0·5, which indicates that half of the current information has succeeded to the next step, while those to R2 are approximately zero, which means it has little influence. Thus, the decomposed characteristics show that the nature of low frequencies of the time-variant AR model has little change

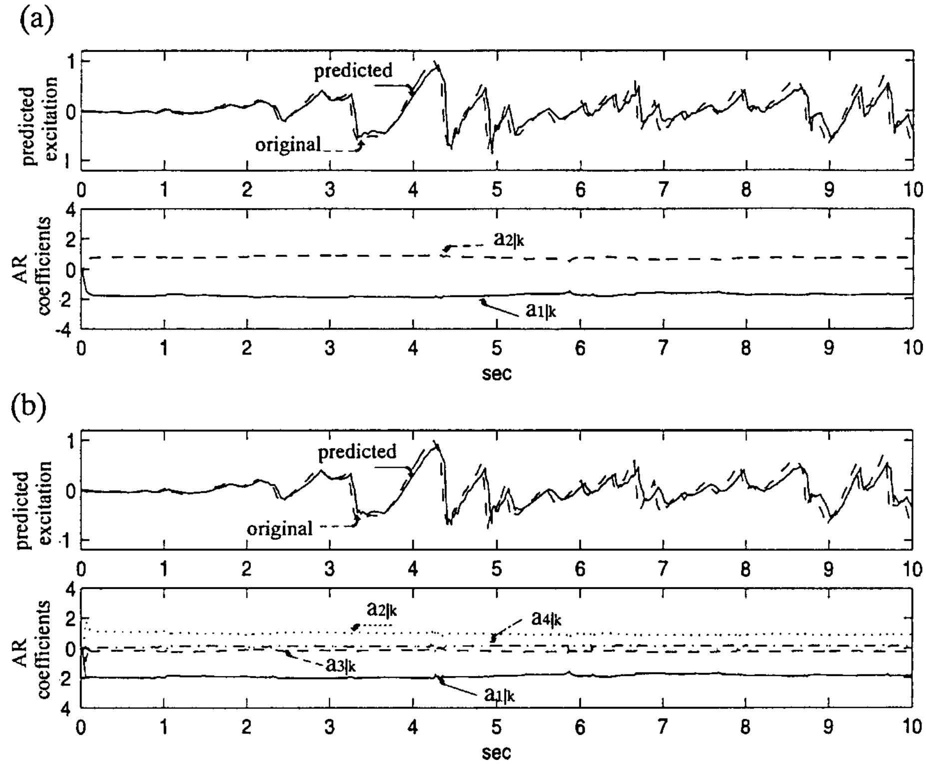

Figure 3. Predicted excitations and AR coefficients for El Centro ($\Delta t = 0{\cdot}01$ s, $l = 100$): (a) for $q = 2$; (b) $q = 4$

despite the change in the AR coefficients. However, the identification errors largely change with time. It is large with respect to the excitation's amplitude. In fact, the predicted identification errors mainly influence the amplitude of the predicted excitation, while the time-variant AR model simply works as a low-pass filter.

Appropriate identification parameters: Although the identification parameters are fixed in the foregoing studies, let us vary them, assuming that $l = 50$, 100 and 150, and $q = 1$–10. Then, estimate their influence, computing the Maximum Identification Error (MIE), the Root Mean Square Prediction Error (RPE) and the Maximum Prediction Error (MPE). MIE means the maximum of $\zeta(k\,|\,k)$ for 10 s, RPE means the root mean square of the difference between the original and the predicted wave using the 0·05-s-prior information for 10 s, and MPE means maximum difference between them. Figure 7 shows these error versus q relations for each l.

For both El Centro and Hachinohe, the larger the q, the less the MIE; and the longer the l, the less the MIE. However, if q is larger than two, the difference is small. RPEs for both El Centro and Hachinohe are least when $q = 2$, and become larger for larger q. Those for $l = 50$ are larger than those for $l = 100$ and 150, while the latter two are almost equal. The MPEs are small for around $q = 2$–4 for both El Centro and Hachinohe. Because a shorter l and a smaller q require less

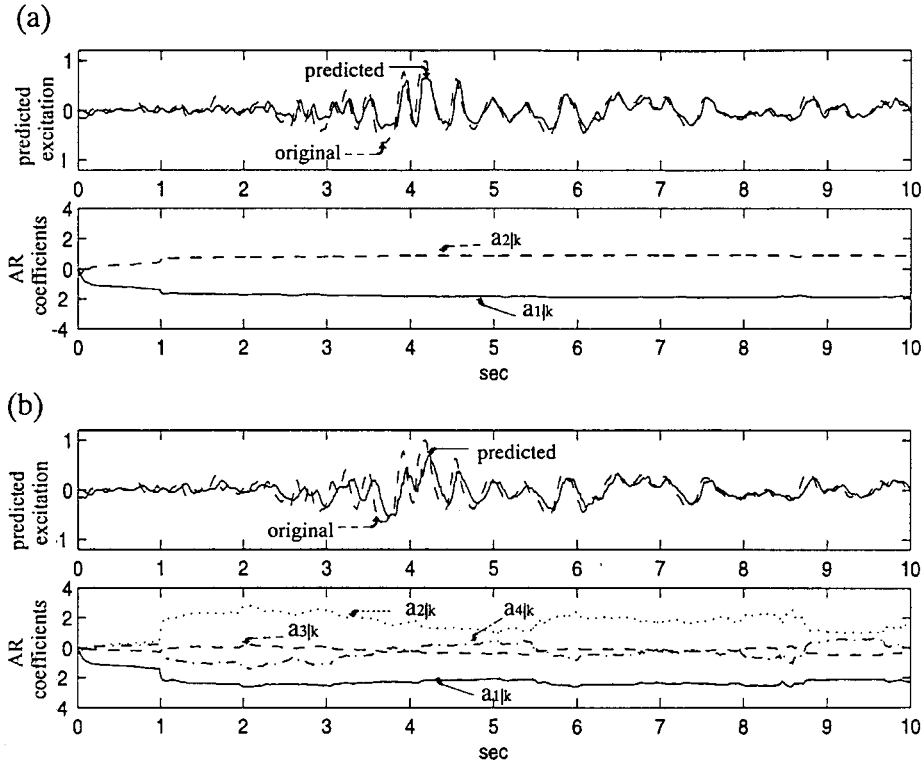

Figure 4. Predicted excitations and AR coefficients for Hachinohe ($\Delta t = 0{\cdot}01$ s, $l = 100$): (a) for $q = 2$; (b) $q = 4$

computation, it is judged that $l = 100$ and $q = 2$–4, which are in fact used in the foregoing studies, are appropriate for a practical use. However, it should be remarked that these parameters are appropriate only if $\Delta t = 0{\cdot}01$ s.

PREDICTION USING FIXED-COEFFICIENT AR MODEL

Fixed-coefficient AR model: In the foregoing studies, the transfer functions and the decomposed forms of time-variant AR models do not change much in low-frequency components despite change in AR coefficients time by time. Thus, we may satisfactorily predict near-future excitation by a fixed-coefficient AR model. Let us examine the performance of a fixed-coefficient AR model, assuming the following models:

Model-F2: $\{a_{1|k}\ a_{2|k}\} = \{-2{\cdot}0\ \ 1{\cdot}0\}$

Model-F4: $\{a_{1|k}\ a_{2|k}\ a_{3|k}\ a_{4|k}\} = \{-1{\cdot}80\ \ 0{\cdot}85\ \ -0{\cdot}20\ \ 0{\cdot}15\}$

where these coefficients are determined on the basis of the foregoing results by the time-variant AR model. We can predict future excitation by updating the preceding excitation information for

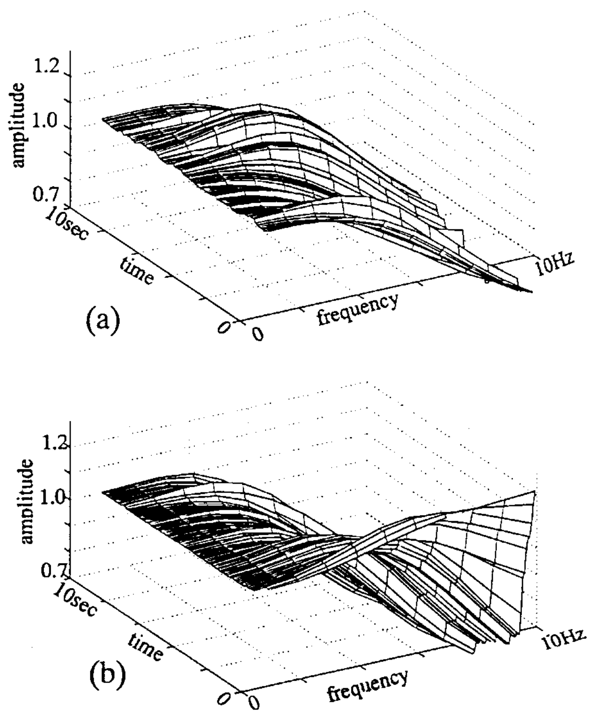

Figure 5. Transfer functions of time-variant AR model for El Centro and Hachinohe ($\Delta t = 0{\cdot}01$ s, $l = 100$, $q = 4$): (a) for El Centro; (b) for Hachinohe

these fixed-coefficient AR models. The prediction using not only 0·05-s-prior information but also 0·10-s-prior information is examined.

Results: Figure 8 shows the transfer functions of the assumed AR models. These transfer functions show characteristics at low frequencies similar to those of the foregoing time-variant AR model. By Model-F2, the higher the frequency, the more the components are filtered. Model-F4 also filters high-frequency components, but maintains the amplitude of 10·0–20·0 Hz components.

Figures 9 and 10 compare the predicted excitations using the 0·05- and 0·10-s-prior information with the original for El Centro and Hachinohe. The predicted excitations by the fixed-coefficient AR model are only slightly inferior to those by the time-variant AR model. The predicted excitations by the fixed-coefficient AR model have spike-like changes for El Centro when the waves have steep slopes. However, those for Hachinohe is closer to the original than those for the time-variant AR model, especially at peaks around 4·0–5·0 s. This is because Models-F2 and -F4 pass more high-frequency components than the time-variant AR model. The difference between Models F2 and F4 is small in prediction using the 0·05-s-prior information. Furthermore, the prediction using 0·10-s-prior information by Model-F2 has a small phase delay from the original and contains too-amplified noise. However, that by Model-F4 is not far from the original. This is because Model-F4 can pass more high-frequency components than Model-F2. It is

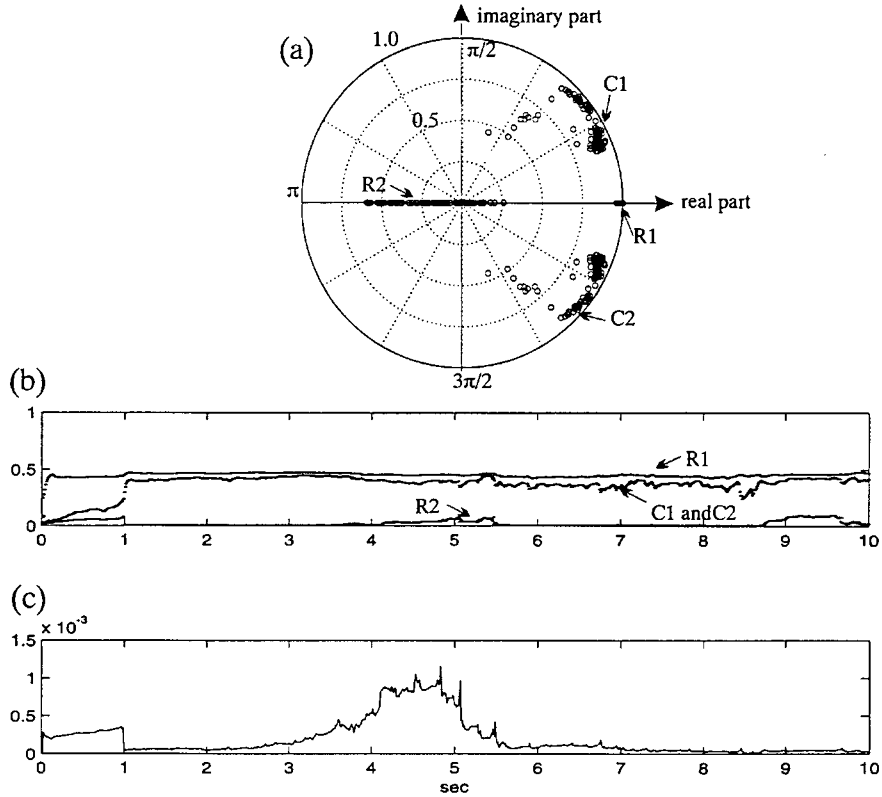

Figure 6. Decomposed characteristics and identification errors for Hachinohe ($\Delta t = 0{\cdot}01$ s, $l = 100$, $q = 4$): (a) eigenvalues of $H_{|k}$, $\lambda_{i|k}$; (b) absolute values of contribution factors for R1, R2, C1 and C2, $f_{i|k}$; (c) identification errors, $\zeta(k|k)$

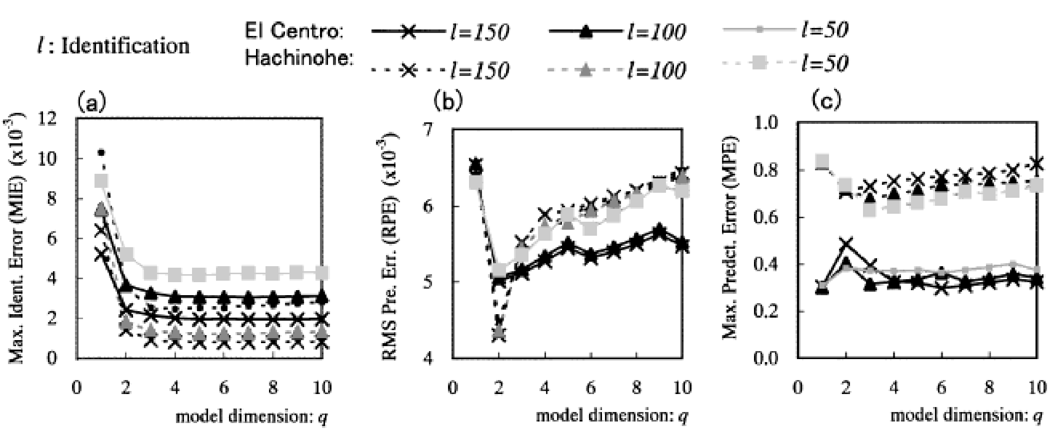

Figure 7. Identification and prediction errors for El Centro and Hachinohe (a) MIE — q; (b) RPE — q; (c) MPE — q

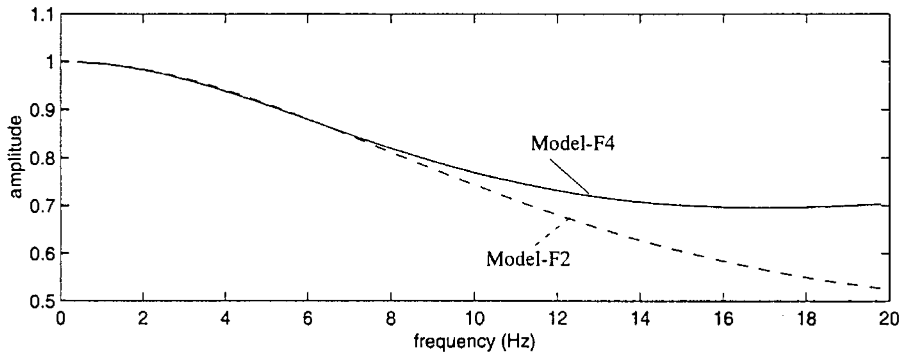

Figure 8. Transfer functions of fixed-coefficient AR models

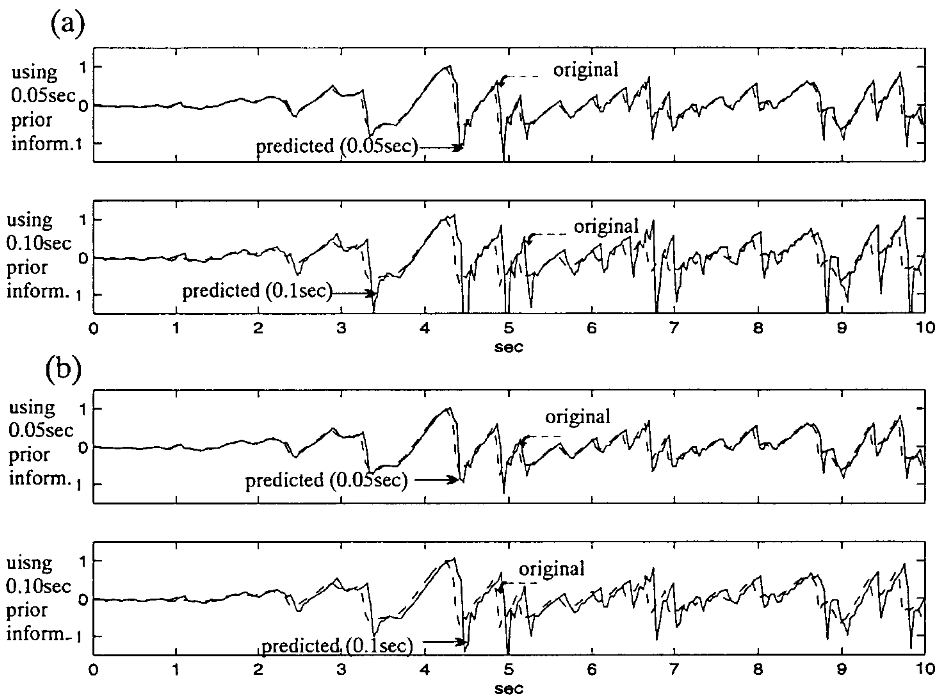

Figure 9. Predicted excitations by fixed coefficient AR models for El Centro (a) for $q = 2$; (b) $q = 4$

judged that Model-F4 gives the best in prediction for the near-future seismic excitation of the assumed cases.

CONCLUSIONS

A real-time prediction method for a near-future seismic excitation using an AR model is examined for future application in an active structural control. The following conclusions are obtained.

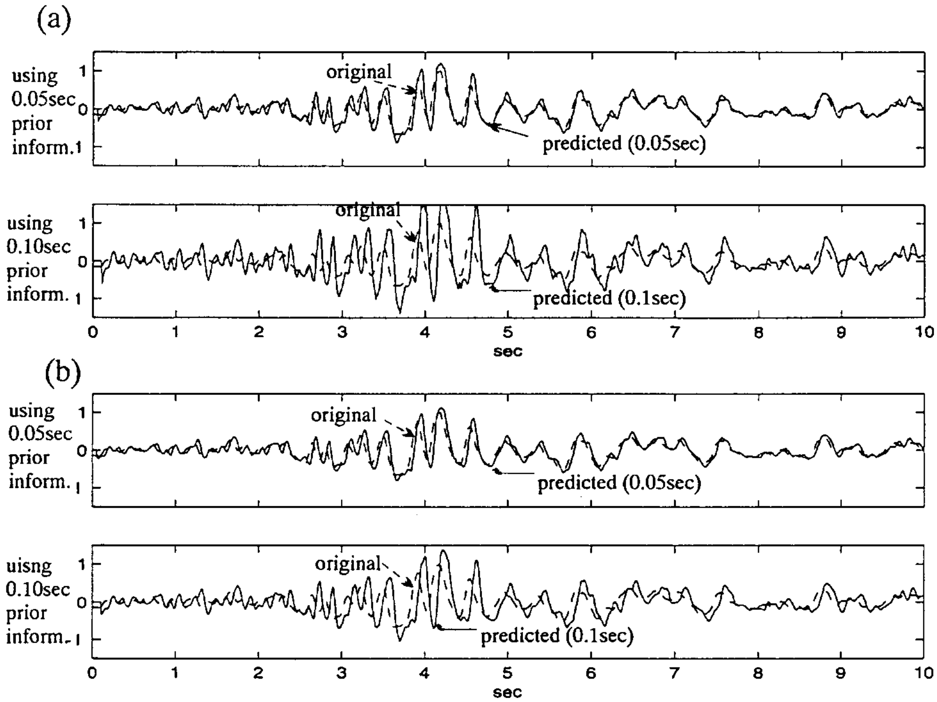

Figure 10. Predicted excitation by fixed coefficient AR models for Hachinohe (a) for $q = 2$; (b) for $q = 4$

(1) A time-variant AR model can predict the 0·05-s-future seismic excitation.
(2) To examine the basic performances of a time-variant AR model, its transfer function and decomposed form are introduced.
(3) Appropriate parameters, i.e. identification time and model dimension for identifying the coefficients of the time-variant AR model are obtained by numerical experiments.
(4) A time-variant AR model simply works as a low pass filter, while the identification error mainly influences the amplitude of the predicted excitation.
(5) The time-variant AR model with appropriate identification parameters produces little change in low-frequency components despite change in AR coefficients.
(6) Even a fixed-coefficient AR model, which has low-frequency characteristics similar to the time-variant AR model, can predict near-future seismic excitation information.

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